

New probe of modified gravity

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We suggest a new efficient way to constrain a certain class of large scale modifications of gravity. We show that the scale-free relation between density and size of Dark Matter halos, predicted within the Λ CDM model with Newtonian gravity, gets modified in a wide class of theories of modified gravity.

Models with the large scale modification of gravity are actively discussed in the recent years in connection with the observed accelerated expansions of the Universe and because they can be related to the existence of extra dimensions [1, 2]. However, the general principles of gauge invariance and unitarity strongly constrain possible theories of gravity, modifying the Newton's law at large distances (see analysis of [3]). Thus, in addition to its phenomenological applications, this problem is related to the fundamental questions of particle physics, field theory and gravity. It is therefore important to search for large scale modifications of gravity experimentally.

A possible set of consistent (as a spin-2 field theory) large scale modifications of gravity is described by two parameters – scale r_c and a number $0 \leq \alpha < 1$ [3–5]. r_c marks the distances at which at the *linearized level* gravitational law changes from $1/r^2$ to some other power $1/r^n$, and the parameter α determines the value of n . Phenomenologically, deviations from Newton's law we are looking for may be represented in this parameter space. A significant fraction of this space is excluded by precision measurements of the Moon orbit [6]. Other natural probes of such modifications are cosmological observables (see e.g. [7–12] and refs. therein).

In this work we identify a new observable sensitive to the large scale modifications of gravity. We demonstrate that universal properties of individual dark matter halos are also affected by the modifications of gravity, and provide novel way to probe them. Namely, we show that the scale-invariant relation between density and size of dark matter halos, predicted by Newtonian gravity within the Λ CDM model [13] and found to hold to a good precision in observed dark matter halos [14], may receive non-universal (size-dependent) corrections for a wide range of parameters r_c and α .

Formation of structures in the Universe is an interplay between gravitational (Jeans) instability and overall Freedman expansion. The gravitational collapse does not start until the potential energy \mathcal{U} of a gravitating dark matter system overpowers the kinetic energy of the Hubble expansion $\mathcal{K} \sim \frac{1}{2}H^2R^2$. Once the gravitational collapse has begun, at any moment of time t a dark halo is confined within a sphere of zero velocity or a *turn-around sphere*. As Hubble expansion rate $H(t)$ decreases with time, the turn-around radius $R_{\text{ta}}(t)$ grows. In the Newtonian cosmology with potential $\phi_N(r) = -GM/r$ the turn-around radius R_{ta} is

$$R_{\text{ta}} \propto \left(\frac{GM}{H^2} \right)^{1/3} \quad (1)$$

(today for masses $\sim 10^{12}M_\odot$ the turn-around radius is \sim

1Mpc). Notice that at any moment of time the average density within a turn-around radius (1) is proportional to the cosmological density and is the same for halos of all masses:

$$\rho_{\text{ta}} \propto \frac{H^2}{G} \propto \bar{\rho}_{\text{tot}}(t) \quad (2)$$

It was shown in [13] that the property (2) leads to a universal relation between characteristic scales and densities of dark matter halos. This relation holds in wide class of dark-matter dominated objects (from dwarf galaxies to galaxy clusters) [14] (see also [15–17]). The relation is in a very good agreement with pure dark matter simulations [18, 19], suggesting that baryonic feedback can be neglected in this case. Therefore, this relation can serve as a new tool of probing properties of dark matter and gravity at large scales.

The relation (2) continues to hold in the Universe where gravity is modified by the cosmological constant Λ . The gravitational energy of a body of mass M at distance r becomes $\mathcal{U}_\Lambda = -\frac{GM}{r} - \frac{\Lambda r^2}{6}$. Comparing it with the kinetic energy of the Hubble flow \mathcal{K} one arrives once again to the relation (2)

$$\rho_{\text{ta}}(t) \propto \frac{\Lambda}{G} \quad (3)$$

(c.f. [13]). The relation (1) still holds and is again independent on the mass of the halo.

What is the most general form of gravitational potential, for which the property (2) remains true? Clearly, it will hold for all the gravitational potentials of the form

$$\phi(r) = -\frac{GM}{r} F\left(\frac{\rho(r)}{\rho_*}\right) \quad (4)$$

where ρ_* is some constant with dimension of density. In particular, Λ -term obeys this property (with $\rho_* \propto \Lambda/G$). All the theories of the form (4) obey the property that relative correction to the Newtonian potential ϕ_N depends *only* on the density $\rho(r)$ within a radius r (and not on the mass or the size of objects).

Next, we consider the modifications of gravity [3, 4]. The gravitational potential of a spherically symmetric system of mass M there has the form

$$\phi_\alpha(r) = -\frac{GM}{r} \pi\left(\frac{r}{r_V}\right) \quad (5)$$

where $0 \leq \alpha < 1$. Here the characteristic (*Vainstein*) radius r_V is defined as [3, 4, 20]

$$r_V = \left(2GM r_c^\beta \right)^{\frac{1}{1+\beta}} \quad \text{where } \beta = 4(1-\alpha) \quad (6)$$

If the scale r_c is of the order of $\sim H_0^{-1}$, such modifications of gravity can provide an explanation for the late-time cosmological expansion of the Universe [1, 2]. The corrections to Newton's law become negligible as $r \rightarrow 0$ ($\pi(0) = 1$) and the radius (6) characterizes the scale where the deviations from Newton's potential become of order unity. Using the relation

$$\frac{r}{r_V} = \left(\frac{r^{1+\beta}}{2GM r_c^\beta} \right)^{\frac{1}{1+\beta}} \propto M^{\frac{\beta-2}{3(1+\beta)}} \frac{1}{\rho^{1/3} (2G r_c^\beta)^{\frac{1}{1+\beta}}} \quad (7)$$

(where $\rho = M/r^3$) we find that among the theories of modified gravity (5) only $\beta = 2$ ($\alpha = \frac{1}{2}$, the DGP model [1]) possess the property (4) and consequently (2).

The property (2) can be probed experimentally. Extensive catalog of DM-dominated objects of all scales, collected in [14], exhibits a simple scaling relation of the properties of the DM halos. Dark matter distribution in the majority of observed objects can be described by one of the universal DM profiles (such as e.g. NFW [21] or Burkert [22]). Such profiles may be parametrized by two numbers, directly related to observations – a characteristic radius r_C (off-center distance where the rotation curve becomes approximately flat, equal e.g. to r_s for NFW) and a DM central mass density $\bar{\rho}_C$, averaged inside a ball with the size r_C . It was shown that DM column density $\mathcal{S} \propto \bar{\rho}_C r_C$, (see [13, 14] for a detailed definition) changes with the mass as $\mathcal{S} \propto M^\kappa$, where $\kappa \approx 0.22 - 0.33$. It was demonstrated in [13] that in the simplest self-similar model (i.e. assuming that r_C/R_{ta} is the same for the DM halos of all masses) property (1–2) implies a scaling $\mathcal{S} \propto M^{1/3}$, compatible with observations. Observations (see e.g. [23–27]) demonstrate that the ratio of r_C to the virial radius depends weakly (as $M^{\sim -0.1}$) on the mass of DM halos. The Fig. 1 shows ratio of the virial radius of a halo to its r_C for DM density profiles from the catalog of [14]. These results are in perfect agreement with the Λ CDM numerical simulations (see e.g. [18, 28]). Due to this slight deviation from the self-similarity, the best fit value of the scaling parameter $\kappa = 0.23$ (see [13] for discussion). For the qualitative discussion of this work, it is important that $\mathcal{S}(M)$ is a featureless power-law dependence, whose slope does not depend on mass and that the deviation from the slope $\frac{1}{3}$ is small (as follows from observations).

We can conclude that in the theories, satisfying the condition (5) (e.g. in DGP model) the properties of DM halos theory will follow the same scaling relation $\mathcal{S} \propto M^{1/3}$ and the difference with the Λ CDM case will only be in a different normalization of this scaling relation. For other theories, described in [3, 4], with $\alpha \neq \frac{1}{2}$, we can see from the Eq. (7)) that the potential $\phi_\alpha(r)$ is not of the form (4) and we can expect deviation from the universal scaling law.

I. $\mathcal{S} - M$ RELATION FOR GENERAL α

Let us work out the $\mathcal{S} - M$ for a general α in details. A general expression, relating the turn-around time, turn-around radius and mass within this radius follows from energy con-

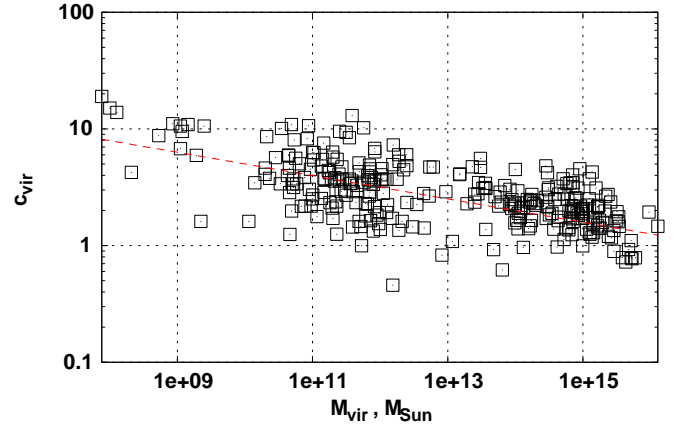


FIG. 1: Comparison of $c_{vir} = R_{vir}/r_C$ as a function of DM halo mass for observed DM density profiles from the catalog of [14].

servation and is given by

$$t_0 = \frac{1}{\sqrt{2}} \int_0^{R_{ta}} \frac{dr}{\sqrt{\phi_\alpha(r) - \phi_\alpha(R_{ta})}} \quad (8)$$

Using the general form (5) we can rewrite the expression (8) in the following form:

$$t_0 = \left(\frac{\pi^2 R_{ta}^3}{8GM} \right)^{1/2} I(x_{ta}) \quad (9)$$

where dimensionless ratio $x_{ta} \equiv R_{ta}/r_V$ and the function $I(x_{ta})$ is given by

$$I(x_{ta}) = \frac{2}{\pi} \int_0^1 \frac{dx}{\left(\frac{\pi(x x_{ta})}{x} - \pi(x_{ta}) \right)^{1/2}} \quad (10)$$

The solution of this equation gives us the “density” $\rho_{ta} \equiv M/R_{ta}^3$ as a function of M . When $r_c \rightarrow \infty$ the turn-around density ρ_{ta} becomes

$$\rho_{ta} = \frac{M}{R_{ta}^3} = \frac{\pi^2}{8Gt_0^2} \equiv \rho_0 \quad (11)$$

Here the *constant* ρ_0 is a function of lifetime of the Universe only and does not depend on parameters of a dark matter halo. This gives a desired relation between a turn-around density and the life-time of the Universe in the pure Newtonian cosmology (without cosmological constant). The function $I(x)$ is defined in such a way that in the Newtonian limit $\pi(x) = 1$ one gets $I(x) = 1$.

The derivation of Eq. (9) demonstrates that for all theories of gravity of the form (4) (including Λ -term and the DGP model) ρ_{ta} is a function of ρ_0 *only* and does not depend on the mass/size of a particular halo. As a result $\mathcal{S} \propto M^{1/3}$ (see [13] for details).

Further analysis of Eq. (9) depends on the form of the function $\pi(x)$. Its exact form is not known (apart from the DGP case). Let us start with analyzing several limiting cases.

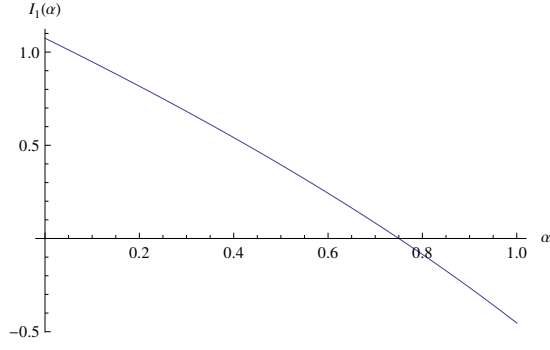


FIG. 2: Function $I_1(\alpha)$ defined in Eq. (13)

If the Vainstein radius $r_V \ll R_{ta}$ for halos of all masses that are experimentally observed (roughly from $\sim 10^8 M_\odot$ to $\sim 10^{16} M_\odot$), then for distances $r \gg r_V$ the corrections to the Newtonian potential reduce either to the order one renormalization of the gravitational constant (on the “normal branch”) or become indistinguishable from the Λ -term (“self-accelerated branch”). In both cases $\mathcal{S}(M) \propto M^{1/3}$ with the normalization, different from the pure Newtonian case.

In the opposite case $r_V \gg R_{ta}$, one can utilize the perturbative expansion of the function $\pi(x)$. The gravitational potential well inside the Vainstein radius is given by [3, 4]

$$\pi(x \ll 1) \approx 1 + c_1 x^a \quad ; \quad a = \frac{\beta + 1}{2} = \frac{5 - 4\alpha}{2} \quad (12)$$

where $c_1 \sim \mathcal{O}(1)$ and is positive for the “normal branch” and negative for the “self-accelerating branch” [5] in full analogy with the DGP model. Notice that $a > 1$ for $\alpha < \frac{3}{4}$. Using expansion (12) one arrives to

$$I(x_{ta} \ll 1) \approx 1 + \frac{c_1}{2} x_{ta}^a \underbrace{\left[\frac{2}{\pi} \int_0^1 dx \frac{1 - x^{a-1}}{\left(\frac{1}{x} - 1\right)^{3/2}} \right]}_{\equiv I_1(\alpha)} \quad (13)$$

where the function $I_1(\alpha)$ is shown on the Fig. 2.

Substituting the expression (13) back into equation (9) and using (7), we obtain

$$\left(\frac{\rho_0}{\rho_{ta}} \right)^{1/2} \left(1 + \frac{c_1}{2} x_{ta}^a I_1(\alpha) \right) = 1 \quad (14)$$

As $x_{ta} \ll 1$ and $a > 1$, one finds that

$$\rho_{ta} \simeq \rho_0 \left(1 + c_1 I_1(\alpha) x_{ta}^a(\rho_0) \right) \quad (15)$$

where to compute x_{ta} we use Eq. (7) with ρ_0 instead of ρ_{ta} . From Eqs. (7) and (15) we see once again that for all $\alpha \neq \frac{1}{2}$ (i.e. $\beta \neq 2$) the turn-around density ρ_{ta} loses its universality and becomes the function of the halo mass M . The turn-around radius $R_{ta}(M)$ is related to ρ_{ta} via $R_{ta}(M) = (M/\rho_{ta})^{1/3}$.

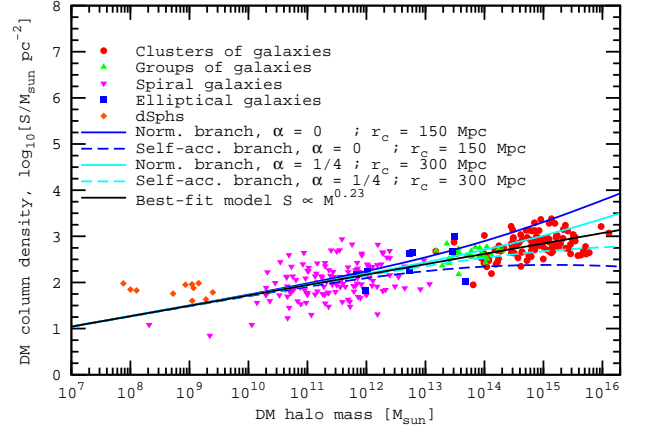


FIG. 3: Examples of $\mathcal{S} - M$ relation in theories with $\alpha < 1/2$ together with the data from [14].

Under the assumption of exact self-similarity, discussed above (i.e. $r_C/R_{ta} = \text{const}$) one arrives to the following expression for \mathcal{S} (recall that $\beta = 4(1 - \alpha)$):

$$\mathcal{S}(M) = \rho_C r_C \propto M^{1/3} \rho_{ta}^{2/3} \quad (16)$$

$$\propto M^{1/3} \rho_0^{2/3} \left(1 + \frac{2}{3} c_1 I_1(\alpha) \left(\frac{M}{M_{lim}} \right)^{\frac{1-2\alpha}{3}} \right) \quad (17)$$

where

$$M_{lim} \equiv \frac{1}{G} \left[\left(\frac{r_c}{2} \right)^{3\beta} \left(\frac{\pi}{t_0} \right)^{2(1+\beta)} \right]^{\frac{1}{\beta-2}} \quad (18)$$

An example of the relation (17) for several α 's and r_c is shown in Fig. 3.

Clearly, the most interesting case is when $r_V \approx R_{ta}$ for some range of observed halo masses. In this regime the deviations from Newtonian gravity become the strongest. The range of values r_c for which this happens is shown in Fig. 4 (the value of t_0 is chosen to be the lifetime of the Universe $t_0 \simeq 1.3 \times 10^{10}$ years). We expect that for r_c in the region Fig.4 the slope of the relation $\mathcal{S} \propto M^\kappa$ will change. Analysis of this case requires however an exact solution of the non-linear analog of the Poisson equation in theories with α (see e.g. [4]), i.e. the knowledge of properties of the function $\pi(r/r_V)$ in the range of radii, where the perturbative expansion (12) breaks down. Notice, that the region where $r_V \approx R_{ta}$ shrinks toward the value $r_c = \frac{2}{\pi} t_0$ as $\alpha \rightarrow \frac{1}{2}$. For this value of r_c the Vainstein radius in the DGP model is equal to the turn-around radius for all halo masses.¹ Knowledge of the potential $\pi(x)$ would also allow to probe the modifications of gravity directly in the Local Group and other nearby galaxies, by studying the infall trajectory around the turn-around radius (see e.g. [29]).

¹ In general for $r_c \sim H_0^{-1}$ the turn-around radius is smaller than r_V for $\alpha < \frac{1}{2}$ and bigger than r_V for $\alpha > \frac{1}{2}$.

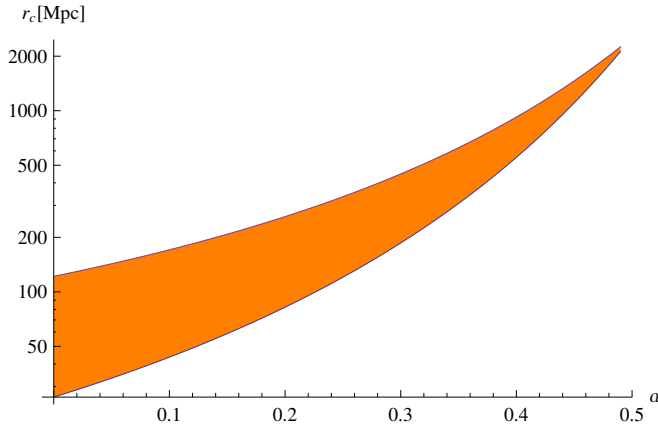


FIG. 4: Range of r_c for which $r_V \approx R_{ta}$ for some halo masses in the range $10^{10} - 10^{15} M_\odot$.

Another way to probe the $S - M$ relation for general r_c and α is to do numerical simulations in the theories of modified gravity (see examples in [11, 12]) and compare directly with observations both the scaling of the central values of S and M and the scatter around it.

Conclusion. The main purpose of this work was to identify a new observable that can be used to constrain the large scale modifications of gravity. We see that the scaling properties of dark matter halos are sensitive to such modifications.

We demonstrated that the models with $\alpha \neq \frac{1}{2}$ predict the deviation from a simple power-law scaling in the $S(M)$ relation. Comparison of predictions of such models with the data, collected in [14] potentially allows to restrict the values of r_c from below for a given α . The improved data-processing and new observational data on DM distributions will allow to strengthen these bounds and make them quantitative.

In this work we have analyzed only the case when the turn-around sphere is well inside the Vainstein radius, $r_V \gg R_{ta}$. To analyze a general case, a better theoretical understanding of the function $\pi(r)$ (defined via Eq. (5)) is needed. Together with better quality of data this will allow to extend our analysis to a wider range of parameters.

In the case $\alpha = \frac{1}{2}$ (the DGP model) the $S(M)$ dependence remains featureless. In this case one has $r_V \approx R_{ta}$ for all masses (for $r_c \sim H_0^{-1}$) and the deviations from the Newtonian gravity at turn-around radius will be strong. Therefore this model (in general, all models that have $r_V \sim R_{ta}$ for halos of $M \sim 10^{12} M_\odot$) can be probed by studying the infall trajectories around the turn-around radius in the Local Group and nearby galaxies [29] using the available data and the data from forthcoming surveys of the Milky Way as well as GAIA mission.

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